

## ***Internal Forces***

ENGR 221  
March 17, 2003

## ***Lecture Goals***

- Internal Force in Structures
  - Axial Load
  - Torque
  - Shear Forces
  - Bending Moment

## ***What use are the Internal Loads?***

In terms of the grand scheme of things, why do we need to find the internal loads?

Stresses are determined by the bending moment and shear stresses are determined by the maximum shear force and the maximum torsional moments.

**NOT ALL MAXIMUM FORCES OCCUR AT THE SUPPORTS!**

## ***General Solution Scheme***

The general scheme for finding the internal set of forces is

- a) Draw the free-body diagram
- b) Determine the support reactions
- c) Apply the equations of equilibrium

$$\begin{array}{lll} \sum F_x = 0 & \sum F_y = 0 & \sum F_z = 0 \\ \sum M_x = 0 & \sum M_y = 0 & \sum M_z = 0 \end{array}$$

## ***Simplest form Solution Scheme***

Axial Forces and Torque in bars and shafts

Internal force act along the axis

Torque acts along the axis of symmetry

From the internal forces we can obtain “axial force diagram” and from the torques we can obtain a “torque diagram” along the axis of symmetry.

## ***Force Diagrams***

Force diagrams show the all of the internal forces acting in the member.

- 1) Axial Force Diagram
- 2) Torque Diagram
- 3) Shear Diagram
- 4) Moment Diagram

### Axial Forces

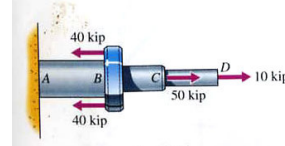
Draw the free-body diagram and solve for the forces acting on the body.

Take sections at each point a transition occurs and solve for the interior forces acting at each point. A transition is a point where a new force is applied. Use equilibrium to solve the problem.

Draw the axial force diagram using the information from the internal forces.

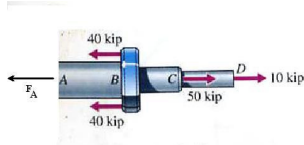
### Example – Axial Load Problem

Determine the axial forces transmitted by transverse cross sections in intervals AB, BC, and CD of the bar. Draw an axial force diagram for the bar.



### Example – Axial Load Problem

Determine the axial forces at point A,

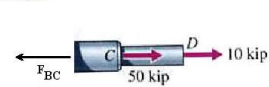


$$\sum F_x = -F_A - 80 \text{ kips} + 50 \text{ kips} + 10 \text{ kips}$$

$$F_A = -20 \text{ kips}$$

### Example – Axial Load Problem

Determine the axial forces at point B,



$$\sum F_x = -F_{BC} + 50 \text{ kips} + 10 \text{ kips}$$

$$F_{BC} = 60 \text{ kips}$$

### Example – Axial Load Problem

Determine the axial forces at point C,

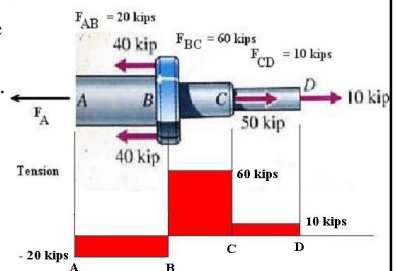


$$\sum F_x = -F_{CD} + 10 \text{ kips}$$

$$F_{CD} = 10 \text{ kips}$$

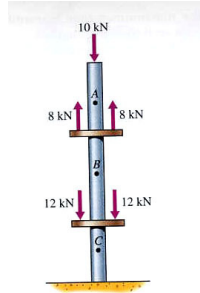
### Example – Axial Load Problem

Draw the axial force diagram, all of the forces are in tension.



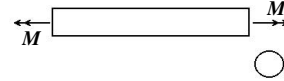
### ***Class – Axial Load Problem***

Determine the axial forces transmitted by transverse cross sections at points A, B, and C of the bar. Draw an axial force diagram for the bar.



### ***Torque***

Torque is a moment acting about the central axis of the body. For a shaft



The double arrow represents the moment at a point

### ***Torque***

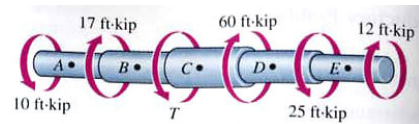
Solve the torque using equilibrium to find all of the moments.

Cut a section at a transition point (torque is applied) and solve the internal moment at the point bring the system back to equilibrium and this is the torque at the point.

Using this information to draw the torque diagram.

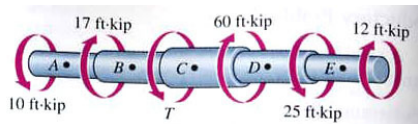
### ***Example – Torque Problem***

For the steel shaft, determine the maximum torque transmitted by any transverse cross section of the shaft. Draw a torque diagram for the shaft.



### ***Example – Torque Problem***

Determine the moment T from equilibrium.

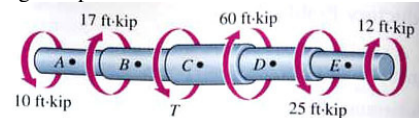


$$\sum M = 10 \text{ k-ft} - 17 \text{ k-ft} + T - 60 \text{ k-ft} + 25 \text{ k-ft} - 12 \text{ k-ft}$$

$$T = 54 \text{ k-ft}$$

### ***Example – Torque Problem***

Determine the moment at point moving from left to the right at point A.

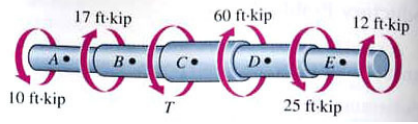


$$\sum M = 10 \text{ k-ft} + M_A$$

$$M_A = -10 \text{ k-ft}$$

### Example – Torque Problem

Determine the moment at point B.

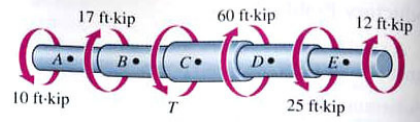


$$\sum M = 10 \text{ k-ft} - 17 \text{ k-ft} + M_B$$

$$M_B = 7 \text{ k-ft}$$

### Example – Torque Problem

Determine the moment at point C.

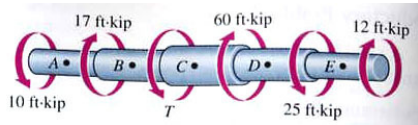


$$\sum M = 10 \text{ k-ft} - 17 \text{ k-ft} + 54 \text{ k-ft} + M_C$$

$$M_C = -47 \text{ k-ft}$$

### Example – Torque Problem

Determine the moment at point D.

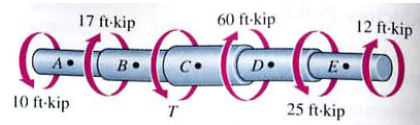


$$\sum M = 10 \text{ k-ft} - 17 \text{ k-ft} + 54 \text{ k-ft} - 60 \text{ k-ft} + M_D$$

$$M_D = 13 \text{ k-ft}$$

### Example – Torque Problem

Determine the moment at point E.

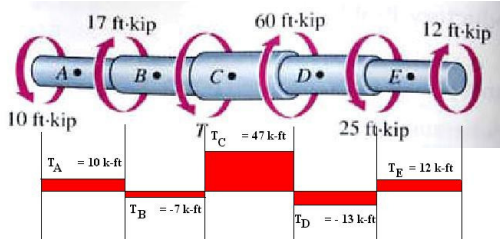


$$\sum M = 10 \text{ k-ft} - 17 \text{ k-ft} + 54 \text{ k-ft} - 60 \text{ k-ft} + 25 \text{ k-ft} + M_E$$

$$M_E = -12 \text{ k-ft}$$

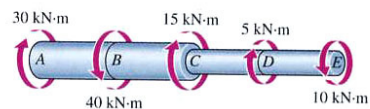
### Example – Torque Problem

Draw the torque diagram



### Class – Torque Problem

For the steel shaft, determine the maximum torque transmitted by any transverse cross section of the shaft. Draw a torque diagram for the shaft.



### ***Internal Forces and Bending Moment***

Three internal force act in members frames and beams, axial load, F, shear force, V, and bending moment, M.

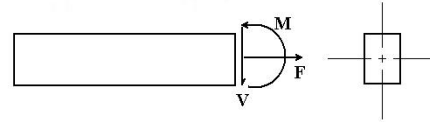
The axial load, F, is in either compression or tension and acts along the centroid of the member.

The shear force, V, acts perpendicular to the surface of the cut or section.

The bending moment, M, acts about the axis coming out of the surface.

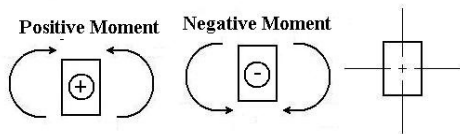
### ***Internal Forces and Bending Moment***

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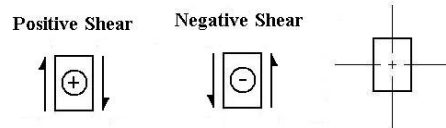
### ***Internal Forces and Bending Moment***

The bending moment, M.



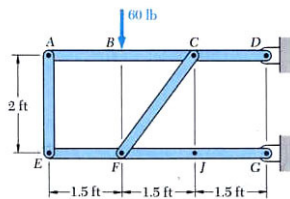
### ***Internal Forces and Bending Moment***

The shear force, V.



### ***Example – Internal Forces in a Frame Problem***

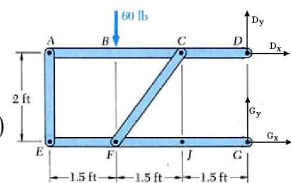
Determine the internal forces in member EFG at point J.



### ***Example – Internal Forces in a Frame Problem***

Draw the free body diagram

$$\begin{aligned} \sum F_x = 0 &= D_x + G_x \\ \sum F_y = 0 &= D_y + G_y - 60 \text{ lb.} \\ &\Rightarrow D_y + G_y = 60 \text{ lb} \\ \sum M_D = 0 &= G_x (2 \text{ ft.}) + (60 \text{ lb})(3 \text{ ft.}) \\ &\Rightarrow G_x = -90 \text{ lb} \\ &\Rightarrow D_x = 90 \text{ lb} \end{aligned}$$



### Example – Internal Forces in a Frame Problem

Look at member EFG

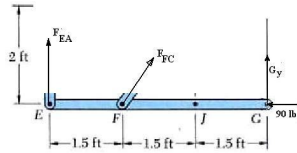
$$\sum F_x = 0 = -90 \text{ lb} + \frac{3}{5} F_{FC}$$

$$\Rightarrow F_{FC} = 150 \text{ lb.}$$

$$\sum F_y = 0 = G_y + \frac{4}{5} F_{FC} + F_{EA}$$

$$\sum M_E = 0 = G_y (4.5 \text{ ft.}) + \frac{4}{5} (150 \text{ lb})(1.5 \text{ ft.})$$

$$\Rightarrow G_y = -40 \text{ lb}$$



### Example – Internal Forces in a Frame Problem

Do a cut at point J and draw a free-body diagram and use equilibrium.

$$\sum F_x = 0 = -90 \text{ lb} + F_{Jx}$$

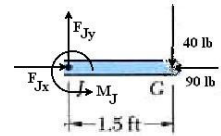
$$\Rightarrow F_{Jx} = 90 \text{ lb.}$$

$$\sum F_y = 0 = -40 \text{ lb.} + F_{Jy}$$

$$\Rightarrow F_{Jy} = 40 \text{ lb.}$$

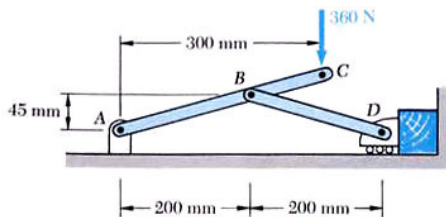
$$\sum M_J = 0 = M_J - (40 \text{ lb})(1.5 \text{ ft.})$$

$$\Rightarrow M_J = 60 \text{ lb-ft}$$



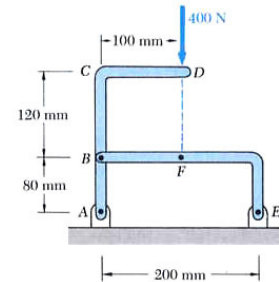
### Quiz

Determine the forces exerted on the wooden block. Points A and D are on the same plane.



### Class – Internal Forces in a Frame Problem

Determine the internal forces in member BE at point F.



### Homework (Due 3/24/03)

Problems:

8-1, 8-6, 8-10, 8-12, 8-14